

Closed-Form Error Analysis on RSS-based Indoor Localization Method

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ABSTRACT

Received Signal Strength (RSS) is considered as a promising measurement for indoor positioning. Lots of RSS-based localization methods have been proposed by its convenience and low cost. This paper focuses on two challenging issues in RSS-based localization schemes: finding optimal localization algorithms and knowing what affects the accuracy of these algorithms. Through theoretical and experimental analysis, we present three important results: 1) we prove that the Non-linear Least Square (NLS) method is efficient to solve the RSS-based localization problem, i.e., its localization error is minimal in theory; 2) we provide a closed-form expression of the localization error for the NLS method. Such an expression reveals the key factors that affect the accuracy of the NLS method; 3) we further conduct a new lower bound of localization error, i.e., the best accuracy achieved possibly. The study of this paper shows the inherent limitation of RSS-based localization schemes and provides guidance for achieving optimal accuracy.

Keywords

Received Signal Strength, Non-linear Least Squares, Indoor Positioning, Closed-Form

1. INTRODUCTION

With the increased demand on location based services (LBS) covering both indoor and outdoor environments, indoor localization technology has achieved significant progress in recent years. Localization schemes based on various measurements such as Time of Arrival (TOA) [1], Time Difference of Arrival (TDOA) [2], Angle of Arrival (AOA) [3], Received Signal Strength (RSS) [4] and others [5] have been proposed. Different from the Global Positioning System (GPS) that uses tens of expensive satellites to provide universal services [6][7], indoor localization systems are sensitive to deploying and maintaining cost since they can only serve in a limited space for local users. Therefore, among

the above mentioned localization schemes, RSS-based localization is a promising approach since the RSS measurement is easy to obtain without extra hardware and software cost. As a matter of fact, beginning in the year 2000, many RSS-based localization methods have been proposed [8].

Basically, RSS-based localization methods should solve two challenging problems: 1) how to find optimal localization algorithms, i.e., the localization error of which is equal to Cramér-Rao lower bound (CRLB); and 2) knowing what factors affect the accuracy of the localization algorithms. Though much work [9][10][11][12][13] has been proposed, the above two challenges have not been solved completely. In one hand, most of the previous studies use simplified assumptions on the range model, e.g., supposing path-loss exponent and transmission power are known. However it is not true in realistic environments. In such a case, localization algorithms optimal for simplified range models may not be efficient for the general case. On the other hand, there is little progress in error analysis on the accuracy of RSS-based localization algorithms. To the best of our knowledge, it is still not clear what factors affect the accuracy of RSS-based localization algorithms exactly.

The goal of this paper is to attempt to solve the above problems, i.e., finding optimal algorithms and knowing what factors affect the localization accuracy. In summary, our main ideas and contributions include:

- We choose the Non-linear Least Squares (NLS) method to address the RSS-based localization problem. Through error analysis we proved that the NLS method is optimal or efficient when the general log-distance path loss range model is adopted. The contribution of this work is to answer the first challenging problem, i.e., the optimal algorithm does exist for RSS-based localization. The existence of optimal algorithms also makes it possible to investigate the exact relationship between key factors of RSS-based localization methods and localization accuracy.
- We analyze the error propagation of the NLS method for RSS-based localization. Using a two-phase conduction, we derive the closed-form expression of localization error for the NLS method. We also derive a new lower bound of the localization error smaller than CRLB. Through of the close-form expression and the lower bound, we find out the actual factors affect the localization accuracy of the NLS method. For example, in this paper, we show, for the first time, that estimation accuracy of path-loss exponent and transmit

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power will affect the localization accuracy. In addition, we also, for the first time, find that the localization accuracy depends on distances among the blind node and anchors.

In summary, we use theoretical and experimental analysis to present the fundamental limitation of RSS-based localization methods, as well as possible ways to improve their accuracy. The remainder of this paper is organized as follows. Section 2 gives the problem and related work. Section 3 analyzes the efficiency of the NLS method based on the general log-distance path-loss range model. Section 4 presents the closed-form analysis on the localization error of the NLS method. Section 5 gives the experimental analysis and Section 6 concludes the paper.

2. THE PROBLEM AND RELATED WORK

In telecommunications and wireless networking, *received signal strength* (RSS) refers to the power level of a signal at the receiver. According to the knowledge of radio propagation, RSS follows the inverse-square law, i.e., the quantity of RSS is inversely proportional to the square of the distance between the transmitter and the receiver [14]. Several range models have been proposed based on the above inverse-square law [15]. Among these models, the one used the most is called the log-distance path-loss model [16][17], which can be described as follows in a 2-D space

$$\bar{P}_i = \alpha - 10\beta \lg \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2} + X_\sigma \quad (1)$$

Here, \bar{P}_i indicates the measured RSS in dBm, which is received at the unknown location (x_u, y_u) (i.e. the blind node) from the anchor (x_i, y_i) . β is the path loss exponent. In a free space, the value of β is 2, which indicates that the power lost is minimal. The signal received at 1 meter is denoted as α , which can be regarded as an indicator of the transmission power. X_σ is the measurement error in \bar{P}_i . Generally, the problem of RSS-based localization can be described as: given m anchors with known positions as (x_i, y_i) and a group of RSS measurements $(\bar{P}_1, \bar{P}_2, \dots, \bar{P}_m)$, how to calculate the location (x_u, y_u) of the blind node?

In fact, the inverse-square law in (1) makes it possible to use the method similar to Multilateration to calculate unknown locations. Therefore, range-based localization methods such as [9][11][18] have been proposed.¹ These methods usually use the range model in (1) to depict the problem of the RSS-based localization. Then, based on the range model, a system of equations can be established. Since the system is non-linear and over-determined, mathematical tools such as Maximum Likelihood Estimation (MLE) and Non-linear Least Squares (NLS) are proposed to solve the system. To avoid the problem of the multiple minima [19], alternative algorithms such as Multidimensional Scaling (MDS) [20], Semidefinite Programming (SDP) [21], Second-Order Cone Programming (SOCP) [22], as well as Linearized Least Squares methods [23][24][25] have been proposed.

¹RSS-based localization methods mainly includes two types of schemes: RSS range-based and range-free. In this paper, we purely focus on RSS aspect, i.e., ignoring the labour and matching effects in Fingerprinting-based localization methods.

Table 1: Classification of RSS-based Localization Methods

Error Expression	Range Models	
	Simplified	General
Non-closed-form	I. [26][27][28][29]	II. [30][31][32]
Closed-form	III. [24][25][33][34][35] [36][37][38]	IV. Our work

Though the above algorithms can solve the range-based localization problem, they can be distinguished by two aspects: 1) Different assumptions on the range model. In fact, the range model in (1) can be classified as the simplified range model and general range model. The former one assumes that parameter β and α , i.e., the path-loss exponent and transmission power, are known, while the later one treats them as unknown parameters that should be calculated. It is obvious that an optimal algorithm based on the simplified range model may not be optimal for the general one. 2) Different expression of the localization error. For RSS-based localization algorithms, the expression of localization error can be classified as closed-form and non-closed-form. The former is preferred because it is possible to analyze the exact relationship between model parameters and the localization error. For the non-closed-form expression, though some analysis can be carried by experiments or simulations, the result may be incomplete due to the diversity of indoor environment and system configurations.

Based upon the above two aspects, the current RSS-based localization methods can be classified into four types, as shown in Table 1. For algorithms in Type I, they did not consider the general case of RSS-based localization problem. In addition, they did not investigate how algorithm parameters affect the localization accuracy. For Type II, though they use the general range model, they can not provide the exact parameter analysis either. For Type III, they can provide the closed-form expression of the localization error. However, they only consider the simplified model and they can not be used for the general case.

Different from previous research, our studies can be classified as Type IV. First, we use the general range model, i.e., regarding the path-loss exponent and transmission power are unknown, as discussed in Section 3. Second, we provide the closed-form expression of the localization error based on the general range model. Based on the closed-form expression, we further present a new lower bound of the localization error to disclose the relationship between localization accuracy and key parameters. Generally speaking, the algorithms in Type I, II and III can be seen as special cases of Type IV.

3. THE RSS-BASED NLS APPROACH

3.1 The General Range Model

In this subsection, we will review the general range model used by the NLS method, i.e., the model used in [30][31]. In fact, denoting a vector θ as $\theta = (x_u, y_u, \beta, \alpha)^T$, here \mathbf{T} indicates the transpose of a vector or matrix, then the range model in (1) can also be expressed in a functional manner

$$\bar{P}_i(\theta) = \alpha - 10\beta \lg \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2} \quad (2)$$

When building a solvable system to calculate the unknown

location of the blind node, (2) is the basic equation. However, as shown in Section 2, different algorithms may use different assumptions on the parameters in (2). In this paper, we choose the general range model which is also adopted in literature such as [30][31] with the following assumptions: 1) We assume X_σ as Gaussian distribution. From the literature, there are different assumptions on X_σ , i.e., the error in \overline{P}_i . In this paper, we choose the assumption adopted by the major research on RSS-based localization, i.e., Gaussian distribution; 2) We assume β and α are unknown parameters to be estimated. In previous studies, different algorithms may treat β and α , or both, as known parameters [39][40][41]. They assume that these parameters can be achieved by calibration or by empirical values. However, just as some studies pointed out [30][31], in practice, these two parameters can not be treated as known values since they would be different in a different space and time, i.e., environment-dependent. Therefore, the general model referred in this paper means that β and α are unknown variables to be estimated.

3.2 Estimating Unknown Parameters

Based on the general range model depicted in (2), in this subsection, we will explain how to apply the NLS method to calculate the unknown parameter $\theta = (x_u, y_u, \beta, \alpha)^T$. First, we need to construct a system of equations with the measured RSS values $(\overline{P}_1, \overline{P}_2, \dots, \overline{P}_m)$

$$\begin{cases} \overline{P}_1 = \alpha - 10\beta \lg \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2} \\ \overline{P}_2 = \alpha - 10\beta \lg \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2} \\ \dots \\ \overline{P}_m = \alpha - 10\beta \lg \sqrt{(x_m - x_u)^2 + (y_m - y_u)^2} \end{cases} \quad (3)$$

Usually, the amount of anchors used in (3) is more than four, i.e., $m \geq 4$. In one hand, to calculate a system with four unknowns, at least four equations are required to find a deterministic solution. On the other hand, since the measurement error existed in \overline{P}_i , more than four measurements or equations are used to eliminate the effects of the random error. It is obvious that the system in (3) is non-linear and over-determined [6][7]. For such a system, we can use the NLS method to find a best solution in the sense of least squares. Basically, the using of the NLS method requires two conditions: 1) the error in \overline{P}_i should be i.i.d, i.e., identical independent distribution; 2) the equations in (2) can be linearized. For the first issue, in the previous subsection, we have shown that the error in \overline{P}_i is assumed as Gaussian, which is i.i.d. For the second issue, later analysis will show that the equations in (3) can be linearized. Then, we define the residual between the measured value \overline{P}_i and the calculated RSS value $\overline{P}_i(\theta)$ as

$$\mathbf{r}_i(\theta) = \overline{P}_i - \overline{P}_i(\theta) \quad (4)$$

Basically, the goal of the NLS method is to find a best approximation of the true θ to minimize the sum of the squares of $\mathbf{r}_i(\theta)$, i.e., minimizing the following

$$\mathbf{S}(\theta) = \sum_{i=1}^m \mathbf{r}_i^2(\theta) \quad (5)$$

In order to find the best approximation, an iterative approach is commonly used to achieve a close approximation by repeatedly refining solutions. Here we use the well-known Gauss-Newton algorithm to explain how to find the solution

in the iterative way. Actually, the Gauss-Newton algorithm uses a following equation to find the approximation of θ iteratively

$$\theta^{(k+1)} = \theta^{(k)} + (\mathbf{J}^{(k)T} \mathbf{J}^{(k)})^{-1} \mathbf{J}^{(k)T} \mathbf{r}(\theta^{(k)}) \quad (6)$$

Here, k indicates the k -th iteration of the calculation, while $k = 0$ indicates that $\theta^{(0)}$ is the initial guess of θ . \mathbf{J} is the Jacobian matrix, whose entries are

$$\mathbf{J}_{ij}^{(k)} = \frac{\partial P_i(\theta^{(k)})}{\partial \theta_j} \quad (7)$$

We can see that through the linearizing in the equation (7), an approximate linear equation for (2) can be found. Based on the linearized equation, a linear system of equations can be built for the non-linear system in (3). Then the iterative process starts from the initial guess $\theta^{(0)}$. The calculation ends when the residual is less than a threshold or other terminating conditions met. When the iteration is finished, the estimated $\theta^{(k+1)}$ will be treated as the true solution θ . Denoting the final Jacobian matrix as \mathbf{J} , we have

$$\mathbf{J} = \begin{pmatrix} -10\beta \frac{x_1 - x_u}{d_1^2 \ln 10} & -10\beta \frac{y_1 - y_u}{d_1^2 \ln 10} & -10 \lg d_1 & 1 \\ -10\beta \frac{x_2 - x_u}{d_2^2 \ln 10} & -10\beta \frac{y_2 - y_u}{d_2^2 \ln 10} & -10 \lg d_2 & 1 \\ \dots & \dots & \dots & \dots \\ -10\beta \frac{x_m - x_u}{d_m^2 \ln 10} & -10\beta \frac{y_m - y_u}{d_m^2 \ln 10} & -10 \lg d_m & 1 \end{pmatrix} \quad (8)$$

where $d_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2}$. In practice, there is no guarantee for the Gauss-Newton algorithm to globally minimize $\mathbf{S}(\theta)$ due to the inherent reasons such as ill-conditioning, divergence, and etc. Thus, a series of techniques were proposed to make the basic Gauss-Newton algorithm more robust².

3.3 The Efficiency of The NLS Estimator

As discussed earlier, when the NLS method is used, we need to know how accurate the method is. From the knowledge of the estimation theory, we know that estimators have a lower bound of the accuracy, i.e., Cramér-Rao lower bound (CRLB). When the accuracy of the estimator is equal to CRLB, the estimator is called efficient. Therefore, in this subsection, we will compare the theoretical error of the NLS method with CRLB to determine if the NLS method is efficient.

First, we present CRLB for the RSS-based localization estimation under the assumption of Gaussian distribution. Considering the equation in (2), i.e.,

$$\overline{P}_i(\theta) = \alpha - 10\beta \lg \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2} \quad (9)$$

Supposing the probability density function (PDF) f of \overline{P}_i is

$$f(\overline{P}_i; \theta) = \frac{1}{\sqrt{2\pi}\sigma_r} \exp\left(-\frac{(\overline{P}_i - \alpha + 10\beta \lg(d_i))^2}{2\sigma_r^2}\right) \quad (10)$$

So the joint PDF of \overline{P}_i , $i = 1, 2, \dots, m$, is

$$f(\overline{P}; \theta) = \prod_{i=1}^m f(\overline{P}_i; \theta) \quad (11)$$

²Among the proposed techniques, we apply the Levenberg-Marquardt method [42], which is widely regarded as the most robust one in dealing with the challenging issues of global convergence.

According to the definition of CRLB, we have

$$C_{\Delta \mathbf{e}} \geq F^{-1} \quad (12)$$

where $F = -E[\nabla(\nabla \ln f(\bar{\mathbf{P}}; \theta))]$. It is easy to get

$$F^{-1} = (\mathbf{J}^T \mathbf{J})^{-1} \sigma_r^2 \quad (13)$$

Next, we need to investigate if $C_{\Delta \mathbf{e}}$ is equal to F^{-1} . If it is, the NLS method will be efficient. Using Taylor series, we can construct a following relationship between θ and $\bar{\theta}$, here $\bar{\theta}$ is the estimation of θ by the NLS method.

$$P_i(\theta) \approx P_i(\bar{\theta}) + \sum_{j=1}^4 \frac{\partial P_i(\theta)}{\partial \theta_j} (\bar{\theta} - \theta) \quad (14)$$

Let $\Delta P_i = P_i(\theta) - P_i(\bar{\theta})$ and $\Delta \theta = \bar{\theta} - \theta$, we have

$$\Delta \theta = (\Delta \theta_1, \Delta \theta_2, \Delta \theta_3, \Delta \theta_4)^T = (\Delta x_u, \Delta y_u, \Delta \beta, \Delta \alpha)^T \quad (15)$$

Then we get

$$\Delta P_i(\Delta \theta) = \sum_{j=1}^4 \frac{\partial P_i(\theta)}{\partial \theta_j} \Delta \theta \quad (16)$$

Denoting $(\Delta P_1, \Delta P_2, \dots, \Delta P_m)^T$ as $\Delta \mathbf{P}$ and denoting $(\Delta \theta_1, \Delta \theta_2, \Delta \theta_3, \Delta \theta_4)^T$ as $\Delta \mathbf{e}$, we get

$$\Delta \mathbf{P} = \mathbf{J} \Delta \mathbf{e} + \epsilon \quad (17)$$

Here, ϵ is the vector of measurement error which can be ignored in practice. It is easy to get

$$\Delta \mathbf{e} = -(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \Delta \mathbf{P} \quad (18)$$

Then the covariance matrix of $\Delta \mathbf{e}$ is

$$C_{\Delta \mathbf{e}} = [(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T] C_{\Delta \mathbf{P}} [(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T]^T \quad (19)$$

Since the covariance matrix of $\Delta \mathbf{P}$ is

$$C_{\Delta \mathbf{P}} = \begin{pmatrix} \text{var}(\Delta P_1) & 0 & \dots & 0 \\ 0 & \text{var}(\Delta P_2) & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \text{var}(\Delta P_m) \end{pmatrix} \quad (20)$$

and

$$\text{var}(\Delta P_i) = \sigma_r^2 \quad (21)$$

Then we have

$$C_{\Delta \mathbf{P}} = \mathbf{I} \sigma_r^2 \quad (22)$$

Here, \mathbf{I} indicates that it is an identity matrix. After simplification, we finally have

$$C_{\Delta \mathbf{e}} = (\mathbf{J}^T \mathbf{J})^{-1} \sigma_r^2 \quad (23)$$

Considering the equation in (13), we have

$$C_{\Delta \mathbf{e}} = F^{-1} \quad (24)$$

and the following theorem:

Theorem 1. The NLS method is an efficient estimator in the case where the RSS measurement error is Gaussian.

Remarks. Theorem 1 shows that the NLS method is an optimal estimator, i.e., has minimal localization error. The result is important due to two reasons: 1) It shows that, for the RSS-based localization problem, there exists algorithms

that can reach the optimal accuracy; 2) It is meaningful to find out the closed-form expression of the error of the RSS-based localization methods. That is, with the closed-form expression, we can exactly know which factors affect the accuracy of the RSS-based localization method.

4. CLOSED-FORM ERROR ANALYSIS

4.1 The Goal and Methodology

Actually, the covariance matrix of $\Delta \mathbf{e}$ can be expressed in another form:

$$C_{\Delta \mathbf{e}} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_\beta^2 & 0 \\ 0 & 0 & 0 & \sigma_\alpha^2 \end{pmatrix} \quad (25)$$

Here, σ_x^2 , σ_y^2 , σ_β^2 and σ_α^2 are the variance of Δx_u , Δy_u , $\Delta \beta$ and $\Delta \alpha$ respectively. Denoting the variance of the localization error as σ^2 , we have

$$\sigma^2 = \sigma_x^2 + \sigma_y^2 \quad (26)$$

From (23), (25) and (26), it is easy to see that the localization accuracy σ^2 depends on $(\mathbf{J}^T \mathbf{J})^{-1}$. However, it is hard to use this matrix form to analyze the relationship between the localization error and parameters contained in the Jacobian matrix in (8).

Therefore, the goal of this subsection is to find the closed-form expression of the localization error σ^2 . We use a two-phase analysis to provide a closed-form expression for the localization error σ^2 : first, we will analyze the relationship between the RSS measurement error and the range estimation; then, we analyze the relationship between the distance estimation error and the final localization error. By this mean, we can finally find the relationship between the RSS measurement error and the localization error.

4.2 Error Analysis on Range Estimation

First, we will determine how measurement error affects the range estimation. Rewriting (2) as a function manner with three unknown parameters

$$P_i(d_i, \alpha, \beta) = \alpha - 10\beta \lg(d_i) \quad (27)$$

It is easy to get

$$d_i(P_i, \alpha, \beta) = 10^{\frac{(\alpha - P_i)}{10\beta}} \quad (28)$$

Denoting the distance calculated by the estimated solutions as $\bar{d}_i = d_i + \Delta d_i$, where d_i is the true distance and Δd_i is the error between \bar{d}_i and d_i . Then we have

$$d_i(\bar{P}_i, \bar{\alpha}, \bar{\beta}) = 10^{\frac{(\bar{\alpha} - \bar{P}_i)}{10\bar{\beta}}} \quad (29)$$

Denoting the variance of the range estimation as

$$\text{var}(d_i + \Delta d_i) = \text{var}(\Delta d_i) = \sigma_{d_i}^2 \quad (30)$$

Denoting the variance of measured \bar{P}_i as σ_r^2 . We also know that the variance of $\bar{\beta}$ and $\bar{\alpha}$ are σ_β^2 and σ_α^2 . It is also easy to obtain the mean of \bar{P}_i , $\bar{\beta}$ and $\bar{\alpha}$ as P_i , β and α respectively. According to the delta method, the approximated variance of the function in (29), i.e., $\sigma_{d_i}^2$ in (30) is

$$\sigma_{d_i}^2 \approx (d_i|_{P_i}(\mathbf{E}[\bar{P}_i]))^2 \sigma_r^2 + (d_i|_{\beta}(\mathbf{E}[\bar{\beta}]))^2 \sigma_\beta^2 + (d_i|_{\alpha}(\mathbf{E}[\bar{\alpha}]))^2 \sigma_\alpha^2 \quad (31)$$

Since

$$(d_i|_{P_i}(\mathbf{E}[\overline{P_i}]))^2 \sigma_r^2 \approx (10^{\frac{(\alpha-P_i)}{10\beta}})^2 (\frac{\ln(10)}{10\beta})^2 \sigma_r^2 \quad (32)$$

$$(d_i|_{\alpha}(\mathbf{E}[\overline{\alpha}]))^2 \sigma_{\alpha}^2 \approx (10^{\frac{(\alpha-P_i)}{10\beta}})^2 (\frac{\ln(10)}{10\beta})^2 \sigma_{\alpha}^2 \quad (33)$$

$$(d_i|_{\beta}(\mathbf{E}[\overline{\beta}]))^2 \sigma_{\beta}^2 \approx (10^{\frac{(\alpha-P_i)}{10\beta}})^2 (\frac{\ln(10)}{10\beta})^2 (10 \lg(d_i))^2 \sigma_{\beta}^2 \quad (34)$$

Let

$$\tilde{\sigma}_i^2 = \sigma_r^2 + \sigma_{\alpha}^2 + (10 \lg(d_i))^2 \sigma_{\beta}^2 \quad (35)$$

We finally have

$$\sigma_{d_i}^2 \approx (10^{\frac{(\alpha-P_i)}{10\beta}})^2 (\frac{\ln(10)}{10\beta})^2 \tilde{\sigma}_i^2 \quad (36)$$

that is

$$\sigma_{d_i}^2 \approx (\frac{\ln(10)}{10\beta})^2 d_i^2 \tilde{\sigma}_i^2 \quad (37)$$

Note. From (37), we can see that the error of the range estimation has a relationship with several factors: the true distance d_i , the true path loss exponent β , the measurement error in $\overline{P_i}$, the estimation error of both β and α . Actually, (37) can be seen as a general form of range estimation. For example, if β and α is perfectly estimated, i.e., $\sigma_{\beta}^2 = \sigma_{\alpha}^2 = 0$, (37) becomes

$$\sigma_{d_i}^2 \approx d_i^2 (\frac{\ln(10)}{10\beta})^2 \sigma_i^2 \quad (38)$$

which is the case described in [36].

4.3 Error Analysis on Location Estimation

Next, we will uncover how range estimation error will affect the localization error. Actually, using estimated parameters $\overline{\alpha}$ and $\overline{\beta}$, we can obtain a group of estimated ranges $\overline{d_i}$ by the equation (28). Then, by $\overline{d_i}$, (x_i, y_i) and (x_u, y_u) , we can build a following system

$$\begin{cases} \overline{d_1} = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2} \\ \overline{d_2} = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2} \\ \dots \\ \overline{d_m} = \sqrt{(x_m - x_u)^2 + (y_m - y_u)^2} \end{cases} \quad (39)$$

Let $\Delta \mathbf{d}$ be the m -length error vector $(\Delta d_1, \Delta d_2, \dots, \Delta d_m)^T$. Denoting $\Delta \mathbf{v}$ as the error vector $(\Delta x_u, \Delta y_u)^T$. Similar to the analysis in Section 3, we have

$$\Delta \mathbf{d} = \hat{\mathbf{J}} \Delta \mathbf{v} + \epsilon \quad (40)$$

where

$$\hat{\mathbf{J}} = \begin{pmatrix} \frac{(x_1 - x_u)}{d_1} & \frac{(y_1 - y_u)}{d_1} \\ \frac{(x_2 - x_u)}{d_2} & \frac{(y_2 - y_u)}{d_2} \\ \dots & \dots \\ \frac{(x_m - x_u)}{d_m} & \frac{(y_m - y_u)}{d_m} \end{pmatrix} \quad (41)$$

and ϵ is the vector of measurement error which can be ignored in practice. By solving the system in (40), we have

$$\Delta \mathbf{v} = -(\hat{\mathbf{J}}^T \hat{\mathbf{J}})^{-1} \hat{\mathbf{J}}^T \Delta \mathbf{d} \quad (42)$$

Denoting $C_{\Delta \mathbf{d}}$ as a $m \times m$ covariance matrix of $\Delta \mathbf{d}$. By the knowledge of the covariance law, $C_{\Delta \mathbf{v}}$ can be calculated by

$$C_{\Delta \mathbf{v}} = (\hat{\mathbf{J}}^T C_{\Delta \mathbf{d}}^{-1} \hat{\mathbf{J}})^{-1} \quad (43)$$

Here $C_{\Delta \mathbf{d}}$ is the covariance matrix of $\Delta \mathbf{d}$, which can be expressed as

$$C_{\Delta \mathbf{d}} = \begin{pmatrix} \sigma_{d_1 d_1}^2 & \sigma_{d_1 d_2}^2 & \dots & \sigma_{d_1 d_m}^2 \\ \sigma_{d_2 d_1}^2 & \sigma_{d_2 d_2}^2 & \dots & \sigma_{d_2 d_m}^2 \\ \dots & \dots & \ddots & \dots \\ \sigma_{d_m d_1}^2 & \sigma_{d_m d_2}^2 & \dots & \sigma_{d_m d_m}^2 \end{pmatrix} \quad (44)$$

Since any two distance estimations are uncorrelated, we have $\sigma_{d_i d_j}^2 = 0$ when $i \neq j$, then we have

$$C_{\Delta \mathbf{d}} = \begin{pmatrix} \sigma_{d_1 d_1}^2 & 0 & \dots & 0 \\ 0 & \sigma_{d_2 d_2}^2 & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \sigma_{d_m d_m}^2 \end{pmatrix} \quad (45)$$

Since

$$\sigma_{d_i d_i}^2 = \sigma_{d_i}^2 \quad (46)$$

and by (37), we have

$$C_{\Delta \mathbf{d}} = (\frac{\ln(10)}{10\beta})^2 \begin{pmatrix} d_1^2 \tilde{\sigma}_1^2 & 0 & \dots & 0 \\ 0 & d_2^2 \tilde{\sigma}_2^2 & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & d_m^2 \tilde{\sigma}_m^2 \end{pmatrix} \quad (47)$$

Putting (47) into (43), we have

$$C_{\Delta \mathbf{v}} = (\frac{\ln(10)}{10\beta})^2 \left(\hat{\mathbf{J}}^T \begin{pmatrix} \frac{1}{d_1^2 \tilde{\sigma}_1^2} & 0 & \dots & 0 \\ 0 & \frac{1}{d_2^2 \tilde{\sigma}_2^2} & \dots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & \frac{1}{d_m^2 \tilde{\sigma}_m^2} \end{pmatrix} \hat{\mathbf{J}} \right)^{-1} \quad (48)$$

Since a multiplication of two 2×2 matrix can be solved by the analytic method, after simplifying, we have

$$C_{\Delta \mathbf{v}} = (\frac{\ln(10)}{10\beta})^2 \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_2 & \psi_3 \end{pmatrix}^{-1} \quad (49)$$

where

$$\psi_1 = \sum_{i=1}^m \frac{(x_i - x_u)^2}{d_i^4 \tilde{\sigma}_i^2} \quad (50)$$

$$\psi_2 = \sum_{i=1}^m \frac{(x_i - x_u)(y_i - y_u)}{d_i^4 \tilde{\sigma}_i^2} \quad (51)$$

$$\psi_3 = \sum_{i=1}^m \frac{(y_i - y_u)^2}{d_i^4 \tilde{\sigma}_i^2} \quad (52)$$

Since we also have

$$C_{\Delta \mathbf{v}} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{pmatrix} \quad (53)$$

We finally get

$$\sigma^2 = \sigma_x^2 + \sigma_y^2 = (\frac{\ln(10)}{10\beta})^2 \frac{\psi_1 + \psi_3}{\psi_1 \psi_3 - \psi_2^2} \quad (54)$$

Then we get the following theorem

Theorem 2. The variance of the NLS method can be expressed as a closed-form expression in (54).

Remarks. From Theorem 2, we can see following facts: 1) the localization error is inversely proportional to the path-loss exponent β , which verifies the result in previous studies; 2) the accuracy has close relationship with the error of range estimation $\tilde{\sigma}_i^2$, i.e., σ_r^2 , σ_β^2 and σ_α^2 ; 3) the accuracy has close relationship with the geometry constructed by anchors and the blind node. More importantly, when β and α are perfectly estimated, i.e., $\sigma_\beta^2 = \sigma_\alpha^2 = 0$, the closed-form expression in (54) is just the expression in [36].

4.4 A New Lower Bound

From Theorem 2, we obtained the closed-form expression of the accuracy of the RSS-based localization. However, the exact relationship between parameters and localization accuracy is not presented except the parameter β . That is, we can not explain how parameters such as $\tilde{\sigma}_i^2$ and the geometry affect the accuracy of the NLS method. Next, we will use the approach of lower bound analysis to investigate how parameters affect the accuracy. Without loss of generality, we assume $x_u = y_u = 0$, i.e., the blind node is located at the origin. Then we have

$$\psi_1 = \sum_{i=1}^m \frac{(x_i - x_u)^2}{d_i^4} = \sum_{i=1}^m \frac{x_i^2}{d_i^4 \tilde{\sigma}_i^2} \quad (55)$$

$$\psi_2 = \sum_{i=1}^m \frac{(x_i - x_u)(y_i - y_u)}{d_i^4} = \sum_{i=1}^m \frac{x_i y_i}{d_i^4 \tilde{\sigma}_i^2} \quad (56)$$

$$\psi_3 = \sum_{i=1}^m \frac{(y_i - y_u)^2}{d_i^4} = \sum_{i=1}^m \frac{y_i^2}{d_i^4 \tilde{\sigma}_i^2} \quad (57)$$

Then $\frac{\psi_1 + \psi_3}{\psi_1 \psi_3 - \psi_2^2}$ can be written as

$$\frac{\sum_{i=1}^m \frac{x_i^2}{d_i^4 \tilde{\sigma}_i^2} + \sum_{i=1}^m \frac{y_i^2}{d_i^4 \tilde{\sigma}_i^2}}{(\sum_{i=1}^m \frac{x_i^2}{d_i^4 \tilde{\sigma}_i^2})(\sum_{i=1}^m \frac{y_i^2}{d_i^4 \tilde{\sigma}_i^2}) - (\sum_{i=1}^m \frac{x_i y_i}{d_i^4 \tilde{\sigma}_i^2})^2} \quad (58)$$

Simplifying (58) we get

$$\frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{(x_i y_j - x_j y_i)^2}{d_i^2 d_j^2 \tilde{\sigma}_i^2 \tilde{\sigma}_j^2}} \quad (59)$$

Denoting the angle of two lines l_i and l_j as ϑ_{ij} , where l_i indicates the line from (x_i, y_i) to $(0, 0)$ and l_j indicate the line from (x_j, y_j) to $(0, 0)$, we get

$$\frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{\sin^2 \vartheta_{ij}}{d_i^2 d_j^2 \tilde{\sigma}_i^2 \tilde{\sigma}_j^2}} \quad (60)$$

Since $\sin^2 \vartheta_{ij} \leq 1$, we have

$$\frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{\sin^2 \vartheta_{ij}}{d_i^2 d_j^2 \tilde{\sigma}_i^2 \tilde{\sigma}_j^2}} \geq \frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{1}{d_i^2 d_j^2 \tilde{\sigma}_i^2 \tilde{\sigma}_j^2}} \quad (61)$$

Denoting the minimal one in d_1, d_2, \dots, d_m as d_{min} and denoting $\sigma_{min}^2 = \sigma_r^2 + \sigma_\alpha^2 + (10 \lg(d_{min}))^2 \sigma_\beta^2$. Since $d_j^2 \tilde{\sigma}_j^2 \geq d_{min}^2 \tilde{\sigma}_{min}^2$, we have

$$\frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{1}{d_i^2 d_j^2 \tilde{\sigma}_i^2 \tilde{\sigma}_j^2}} \geq \frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2 d_{min}^2 \tilde{\sigma}_{min}^2}} \quad (62)$$

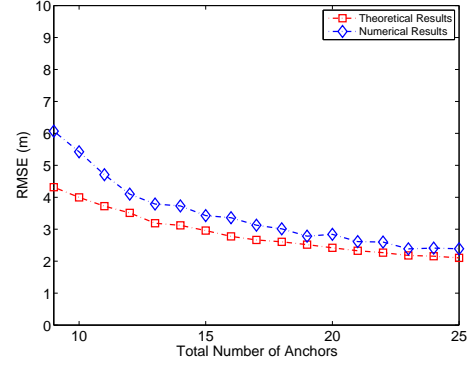


Figure 1: Effects of More Anchors

Since there is

$$\frac{\sum_{i=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2}}{\frac{1}{2} \sum_{i=1, j=1}^m \frac{1}{d_i^2 \tilde{\sigma}_i^2 d_{min}^2 \tilde{\sigma}_{min}^2}} = \frac{d_{min}^2 \tilde{\sigma}_{min}^2 \sum_{i=1}^m \frac{1}{d_i^2}}{\frac{1}{2} m \sum_{i=1}^m \frac{1}{d_i^2}} \quad (63)$$

and

$$\frac{d_{min}^2 \tilde{\sigma}_{min}^2 \sum_{i=1}^m \frac{1}{d_i^2}}{\frac{1}{2} m \sum_{i=1}^m \frac{1}{d_i^2}} = \frac{2}{m} d_{min}^2 \tilde{\sigma}_{min}^2 \quad (64)$$

we finally get

$$\frac{\psi_1 + \psi_3}{\psi_1 \psi_3 - \psi_2^2} \geq \frac{2}{m} d_{min}^2 \tilde{\sigma}_{min}^2 \quad (65)$$

Then, putting (65) into (54), we get the following inequality

$$\sigma^2 \geq \left(\frac{\ln(10)}{10\beta}\right)^2 \frac{2}{m} d_{min}^2 \tilde{\sigma}_{min}^2 \quad (66)$$

and a following theorem:

Theorem 3. The variance of the NLS method satisfies the inequality in (66).

Remarks. From (66), we can see that the lower bound of the RSS-based localization error σ^2 is proportional to d_{min}^2 , $\tilde{\sigma}_{min}^2$, the inverse of m and the inverse of β^2 . Since we also have $\sigma_{min}^2 = \sigma_r^2 + \sigma_\alpha^2 + (10 \lg(d_{min}))^2 \sigma_\beta^2$, then we can see that the lower bound of the localization error is also proportional to σ_r^2 , σ_α^2 and σ_β^2 . The lower bound indicates the inherent limitation of the RSS-based localization schemes, whose accuracy depends on d_{min} . On the other hand, the lower bound also provides guidance to achieve optimal solutions. That is, it is possible to get better localization accuracy by choosing smaller d_{min} or more anchors (increasing m). In fact, our later experiments verify the feasibility of these optimizations.

4.5 Numerical Results

To preliminarily confirm the remarks, we firstly conduct numerical verification before those experiments in real environment. In verification setting, we give a bounded rectangular area with corners $(-50, -50)$, $(-50, 50)$, $(50, 50)$ and $(50, -50)$, and set $\alpha = -25\text{dbm}$, $\beta = 2$. Let m vary from 9 to 25, and we randomly generate the positions of m anchors for 1000 times. In each time, the measurement errors are randomly generated with $\sigma_r = 2$, $\sigma_\alpha = 3$ and $\sigma_\beta = 0.1$. By this mean, we can use the equation in 1 to generate a set of RSS measurements for calculating the unknown location.

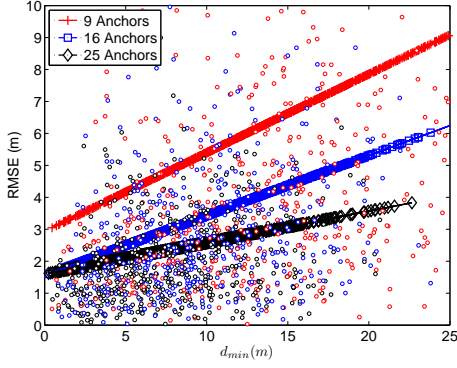


Figure 2: Effects of Larger d_{min}

In Fig. 1, we show the relationship between the total number of anchors and the average localization error, and compare the numerical results with the error calculated by (54) or (59). We can see that the localization error is proportional to the inverse of m . In addition, the two errors are close to each other when the number of anchors increase. Fig. 2 shows the results of the localization error relating to d_{min} ; as we can see from the mean errors of $m = 9$, $m = 16$ and $m = 25$ for each d_{min} , the localization accuracy is probably better when having smaller d_{min} , and a larger m has higher probability to obtain lower localization error.

5. EXPERIMENTS AND VERIFICATIONS

5.1 Experimental Setting

In our experiments, we choose a free space without pillars and other obstructions, which is a 10 meter wide and 10 meter long area as the testing environment. The coordinates of four corners are $(-5, -5)$, $(-5, 5)$, $(5, 5)$ and $(5, -5)$ respectively. The unit of the coordinate system is the meter. Instead of deploying many real anchors, we use the following method to emulate localizing a stationary blind node by a group of anchors. We first use a WiFi hotspot located at $(0, 0)$ as the blind node which broadcasts RSS signals periodically. Then a mobile robot carrying on a cell phone collects RSS signals from the WiFi hotspot automatically according to a planned route. The moving cell phone acts as multiple anchors in different locations. During the movement, the mobile robot will stop for 10 seconds by each 0.05 meter. During the stop, the mobile phone will collect 5 individual RSS signals sent from the WiFi hotspot. The 5 individual signals will be averaged to eliminate possible random measurement errors. Based on the data collected by the above procedure, we can customize the number of RSS signals and placement of anchors.

5.2 Practical Accuracy

5.2.1 Verification Method

In this subsection, we will verify the results of Theorem 2 and 3, i.e., how key factors affect the accuracy of the NLS method. As we know from previous sections, adjustable parameters include the geometry of the anchors, the density of anchors m and the shortest range d_{min} among the blind node and anchors. In this experiment, we mainly consider

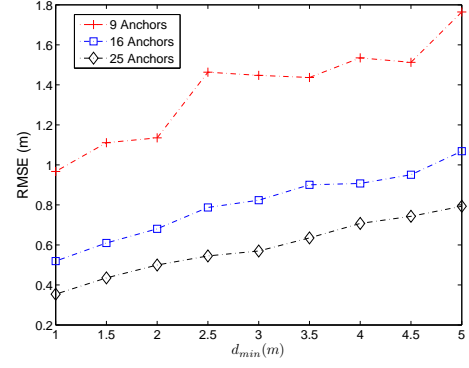


Figure 3: Localization Accuracy

how m and d_{min} affect the localization accuracy, that is, we choose random geometry of anchors in all experiments. For d_{min} , we adjust it from 1 meter to 5 meter. For m , we adjust it by 9 anchors, 16 anchors and 25 anchors respectively. The calculation will be repeated more than 1000 times. Then we use the average as the estimated $\bar{\theta}$.

5.2.2 Result Analysis on Location Estimation

Fig. 3 shows the standard variance or RMSE (Root Mean Squared Error) of the NLS localization method, i.e., σ . From the figure, we can see two phenomena: 1) With the increased d_{min} , the localization error will increase proportionally. It is seen that σ is approximately directly proportional to d_{min} ; 2) It is also obvious that when the amount of anchors increases, the localization error will decrease. This verifies the result of Theorem 3, i.e., the localization error σ is inversely proportional to the square root of the amount of anchors m .

5.2.3 Result Analysis on β and α Estimation

Fig. 4 to Fig. 7 shows the result of estimating β and α and their variance. Fig. 4 and Fig. 5 show the result of estimating the absolute value of β and α . From the figures, we can see that with the increase of d_{min} , the average of β and α are both decreased. Combined with the verification of localization accuracy in Fig. 3, we can see that when d_{min} is smallest (i.e. 1 meter in our experiments), the localization error is minimal, as well as β and α is optimal estimation. This also supports the analysis in Theorem 2 that the larger value of β will bring smaller localization error.

Fig. 6 and Fig. 7 show the standard variance of estimated β and α , i.e., σ_β and σ_α . From the figures, we can see that the estimation errors of β and α have no direct relationship with the value of d_{min} . However, the standard variance is smaller when more anchors are used, which indicates that the estimation of β and α are more accurate if more anchors are used. That is, placing anchors closer to the unknown location will not improve the accuracy of estimating β and α , though increasing its amount can improve the precise of β and α .

5.3 Comparisons

5.3.1 Verification Method

In this subsection, we will compare practical accuracy with theoretical accuracy to verify the correctness of our theoretical analysis. Specifically, we will compare three types of

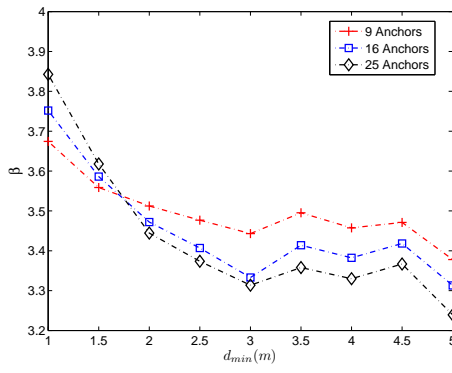


Figure 4: Estimated β

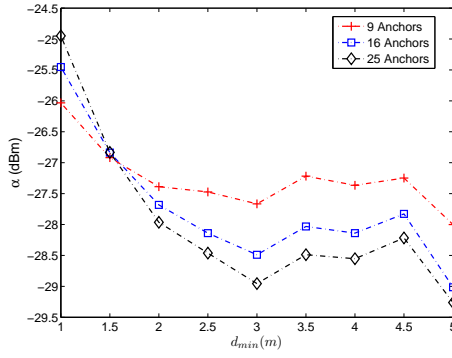


Figure 5: Estimated α

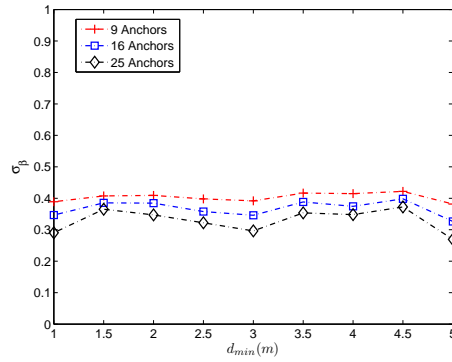


Figure 6: The Accuracy of Estimated β

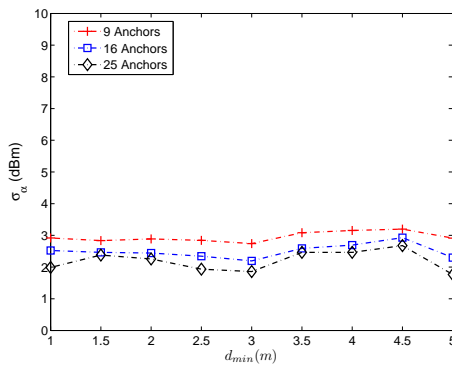


Figure 7: The Accuracy of Estimated α

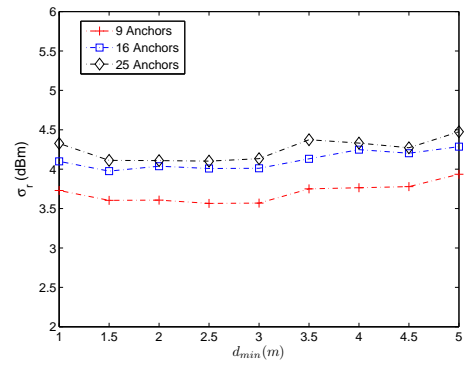


Figure 8: RSS Measurement Error

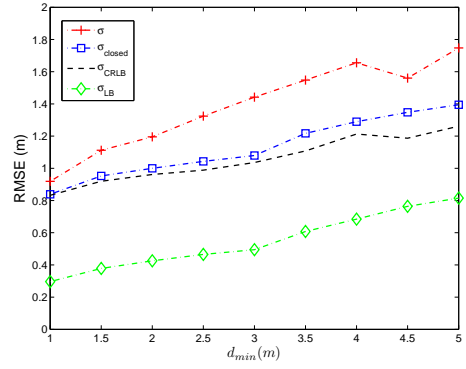


Figure 9: Error Comparisons with 9 Anchors

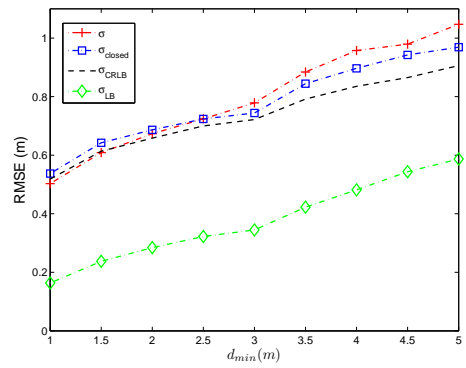


Figure 10: Error Comparisons with 16 Anchors

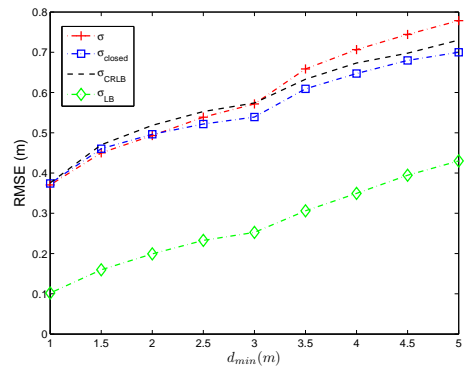


Figure 11: Error Comparisons with 25 Anchors

localization errors: the error calculated by the NLS method σ , the predicted CRLB σ_{CRLB} calculated by (13) and the predicted error σ_{closed} calculated by the closed-form expression in (54). In addition, we also compare these errors with the lower bound σ_{LB} in (66) to see the possible improvement space. Here, σ is the real error of the NLS method. When calculating σ_{CRLB} , σ_{closed} and σ_{LB} , we need know $\theta = (x_u, y_u, \beta, \alpha)$, σ_β^2 , σ_α^2 and σ_r^2 . Then, we use the estimated $\bar{\theta}$ as θ . We also use σ_β^2 , σ_α^2 calculated in Section 5.2. For X_σ or σ_r^2 , we rewrite the equation in (1) as

$$X_\sigma = \bar{P}_i - \alpha + 10\beta \lg(\sqrt{(x_i - x_u)^2 + (y_i - y_u)^2}) \quad (67)$$

Then by a group of $(\bar{\theta}, \bar{P}_i, x_i, y_i)$, we can get an approximated σ_r^2 . Fig. 8 gives the value of σ_r with different d_{min} and the density of anchors. From the figure, we can see that d_{min} does not affect the variance of σ_r , while the density of anchors affects the σ_r significantly.

Since the correctness of the theoretical analysis depends on the accurate estimation of parameters, we choose their values with the optimal conditions, i.e., with the minimal d_{min} and densest of anchors.

5.3.2 Comparisons

Fig. 9, Fig. 10 and Fig. 11 show the results of comparisons when the number of anchors changes. From the figures, we can observe following phenomena:

- When the density of anchors increases, all errors (both estimated and theoretical errors) decrease. This verifies the analysis in Section 4, that is, as the same as the experiments in Fig. 1. Similarly, the relationship between d_{min} and localization error is also same to the results shown in Fig. 3.
- From all figures, we can see that two theoretical errors σ_{CRLB} and σ_{closed} are very close to each other. This verifies that CRLB in (13) is same to the closed-form expression in (54). This also support our analysis on the theoretical accuracy of the NLS method. That is, our analysis is correct because different formula provide the same results.
- Both two theoretical errors, i.e., σ_{CRLB} and σ_{closed} , are close to the estimated error σ , especially when denser anchors are used. The results show that the practical accuracy of the NLS method is close to CRLB and the closed-form error. This also verifies that the NLS method is efficient or can achieve the optimal accuracy.
- Comparing the three errors σ_{CRLB} , σ_{closed} and σ with the lower bound σ_{LB} , we can see that the lower bound is significantly smaller. This indicates that if better geometry of anchors are selected, the accuracy of the NLS method can be further improved.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we focus on solving two challenging issues of RSS-based localization: finding an optimal localization algorithm and key factors affecting the accuracy of RSS-based localization method. Through two-phase closed-form analysis, we show that the Non-linear Least Square method is an optimal algorithm which can achieve CRLB. We also found

key factors affecting localization accuracy uncovered before. Most importantly, our work presents the fundamental limitations of RSS-based localization methods and possible optimizing solutions. In the future, we will continue to investigate key issues related to RSS-based localization. Specifically, we will focus on following issues in later research: 1) Building a larger testing area and deploying WiFi beacons to construct a practical WiFi localization environment; 2) Investigating different types of anchors' geometry to find possible ways to further optimize the accuracy.

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